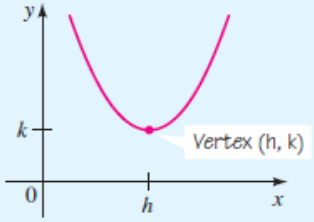
2.5 Quadratic Functions; Maxima and Minima

Standard Form of a Quadratic Function

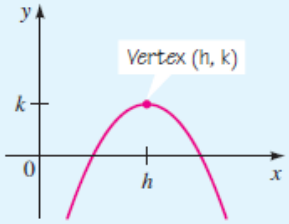
A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k); the parabola opens upward if $a > 0$ or downward if $a < 0$.



$f(x) = a(x - h)^2 + k,\ a > 0$

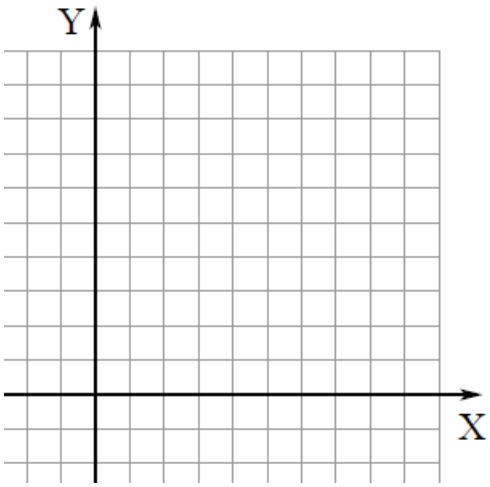


$f(x) = a(x - h)^2 + k,\ a < 0$

The graph of a quadratic is called a **parabola**.

Ex. Standard Form of a Quadratic Function

For the graph of the quadratic function $f(x) = 3x^2 - 6x + 10$, find the coordinates of the vertex and the y-intercept. Then graph the parabola.

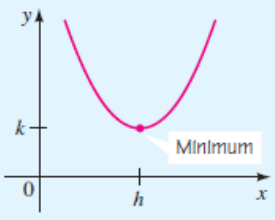


Maximum or Minimum Value of a Quadratic Function

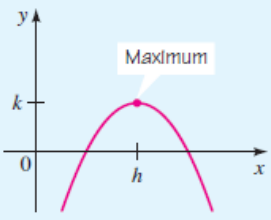
Let f be a quadratic function with standard form $f(x) = a(x - h)^2 + k$. The maximum or minimum value of f occurs at $x = h$.

If $a > 0$, then the **minimum value** of f is $f(h) = k$.

If $a < 0$, then the **maximum value** of f is $f(h) = k$.



$f(x) = a(x - h)^2 + k, a > 0$



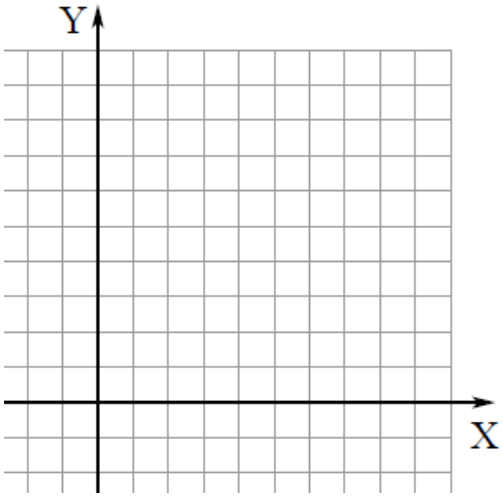
$f(x) = a(x - h)^2 + k, a < 0$

2.5 Quadratic Functions; Maxima and Minima

Ex. Minimum Value of a Quadratic Function

Consider the quadratic function $f(x) = 2x^2 - 20x + 53$.

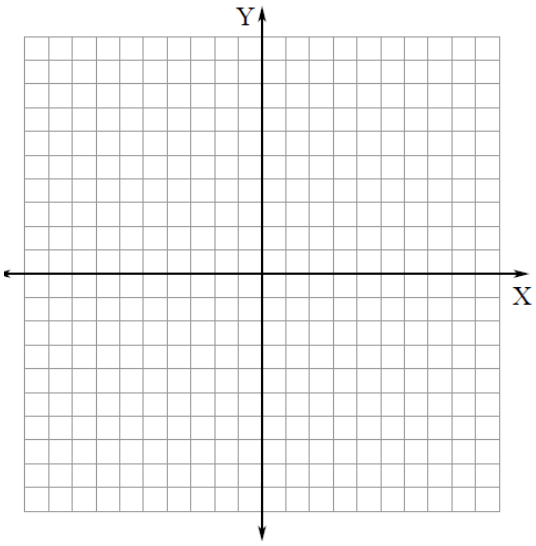
a) Express f in standard form.
b) Sketch the graph.
c) Find the minimum value of f.



Ex. Maximum Value of a Quadratic Function.

Consider the quadratic function $f(x) = -x^2 + x + 5$.

a) Express f in standard form.
b) Sketch the graph of f.
c) Find the maximum value of f.



2.5 Quadratic Functions; Maxima and Minima

Maximum or Minimum Value of a Quadratic Function

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a}$$

If $a > 0$, then the **minimum value** is $f\left(-\frac{b}{2a}\right)$.

If $a < 0$, then the **maximum value** is $f\left(-\frac{b}{2a}\right)$.

Ex. Finding Maximum and Minimum Values of Quadratic Functions

Find the maximum or minimum value of the quadratic function:

a) $f(x) = x^2 + 8x$

b) $g(x) = -4x^2 + 8x - 2$

Ex. Maximum Gas Mileage for a Car

Most cars get their best gas mileage when traveling at a relatively modest speed. The gas mileage M for a certain new car is modeled by the function $M(x) = -\frac{1}{32}s^2 + 3s - 41$, $15 \leq s \leq 70$ where s is the speed in miles per gallon. What is the car's best gas mileage, and at what speed is it attained?

Ex. Finding Local Maxima and Minima from a Graph

Find the local maximum and minimum values of the function $f(x) = x^3 - 10x^2 + x + 4$ correct to three decimals.

Ex. A Model for Maximizing Profit

The profit, in millions of dollars, a factory makes by producing x thousand items is approximated by the function $f(x) = -\frac{x^6}{1000} + x^2$ when x is between 0 and 10 thousand. Estimate the amount of items that the factory should make to maximize its profit.